When working with conservation of energy problems, there are a lot of different energies to consider, so it may seem daunting to figure out where to start. However, most of these problems start with the statement of the conservation of energy and that work done on a system results in a change in energy, or

In fact, there some problems where you’ll have to implement both ideas to solve them, where you can use the conservation of energy to solve as if there were no non-conservative forces, and then figure out how the energy changes with the addition of these forces.

When working with mechanical energies, the two that are mostly used are gravitational potential and kinetic, or:

The h in gravitational potential will be relative to where we put h=0 to be, which can be anywhere. Usually, it’s a good idea to put h=0 in a position that makes either the final energy or the initial energy have zero gravitational potential, as this creates one less energy you need to work out.

Energy is not a vector, so there’s no direction associated with energy, although you can still have positive and negative energies. Notice that the velocity in kinetic energy is always squared. Since mass is always positive, and any value squared will always be positive, kinetic energy will also always be positive, and if you’re solving for velocity given a kinetic energy, the velocity will always be positive. This, however, does not mean that the velocity will always be in the positive direction. The velocity in kinetic energy is the *magnitude* of the velocity, so you will need to figure out what the direction of the velocity.

Conceptual Example

A car sits at rest at the top of a hill. A small push sends it rolling it down a hill. After its height has dropped by 5.0 m, it is moving at a good clip. Write down the equation for conservation of energy, noting the initial and final state, and what energy transformation has taken place.

**Solution**

Since the only force acting on the car is gravity, and gravitational force is conservative, we can use the conservation of energy:

Before we move on to calculating the values for these energies, let’s just think about the energy of the car in both the initial and the final state. In the initial state, the car sits at rest at the top of the hill, so that means it’s velocity is 0 at the beginning, and therefore it has no kinetic energy. In the final state, the car does have some velocity, which means that it has some kinetic energy, and it dropped a few meters, so that means that its gravitational potential has decreased. If we set the final point the car reaches to be at h=0, we can say that the car has zero gravitational potential in its final position and some gravitational potential in its initial position.

Now we can go back to our equations. We can expand our energies to be:

Earlier, we found that Ki is 0 and Uf is 0, so we’re left with

Substituting in our definitions for kinetic energy and gravitational potential energy gives us

We can simplify this even further; notice that we can cancel mass out on both sides of the equation. Doing this and solving for vf gives us

Since we set the height to be zero in the final position, we know the height in the initial is 5 m above that, so that makes hi 5 m. Solving for vf gives us

We can see that the energy transformation here is gravitational potential energy into kinetic energy.

Kingda Ka

The Kingda Ka is a giant rollercoaster located in New Jersey. The ride includes a vertical drop of 127 m. Suppose the coaster has a speed of 6 m/s at the top of the drop. Find the speed of the riders at the bottom, ignoring friction and air resistance.

**Solution**

Again, since there are no non-conservative forces in play here, we can use the conservation of energy here:

Expanding these into potential and kinetic energies gives us:

Since the coaster is moving at both the top and bottom of the drop, we know it has both initial and final kinetic energy. If we set the bottom of the drop to be h=0, we can say that the top has some gravitational potential and the bottom has no gravitational potential. So our new equation becomes:

Expanding these terms out

The mass cancels out on both sides, leaving us with

Solving for the final velocity and putting in our values gives us

Wrecking Ball

A wrecking ball is suspended from a 6.5 m long cable that makes a 45 degree angle with the vertical. The ball is released and swings down. What is the ball’s speed at the lowest point?

**Solution**

No non-conservative forces here, so we can start with conservation of energy again, and expand it out.

The wrecking ball is not moving at the top, but is moving at the lowest part of its swing, so it has no initial kinetic energy but some final kinetic energy. If we set h=0 at the lowest part of its swing, we can say that there’s no final potential gravitational energy, but some initial gravitational potential energy. So our new equation is:

Substituting in our definitions for these energies:

Canceling the mass on both sides and solving for vf:

Fortunately, the mass canceled out, so we don’t need to know the mass of the wrecking ball. However, we do need to know its initial height. Here, it’s helpful to draw a picture. Let’s create a picture with the wrecking ball in both its initial position and its final position

As you can see, the difference in the y distance from the top of the cable to the wrecking ball in both positions will be our hi. For the final position, the y distance is just the length of the cable, 6.5 m. For the initial position, we can use the angle the cable makes with the vertical and the length of the cable to solve for the y distance:

The difference between these is 6.5 m – 3.41 m, which is 3.09 m, our initial height. Now we can plug this into our equation for final velocity: